(4)

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Also, $(\alpha-\delta)\alpha + (\alpha-\delta)(\alpha+\delta) + \alpha\cdot(\alpha+\delta) = \frac{37}{2}$

$\Rightarrow \alpha^2 - \cancel{\alpha\delta} + \alpha^2 - \delta^2 + \alpha^2 + \cancel{\alpha\delta} = \frac{37}{2}$

$\Rightarrow 3\alpha^2 - \delta^2 = \frac{37}{2} \Rightarrow 3\cdot\frac{25}{4} - \delta^2 = \frac{37}{2}$

$\Rightarrow \delta^2 = \frac{75}{4} - \frac{37}{2} = \frac{1}{4}$ $\therefore \delta = \pm\frac{1}{2}$

$\therefore$ The roots are $\frac{5}{2}-\frac{1}{2}, \frac{5}{2}, \frac{5}{2}+\frac{1}{2}$

i.e. $2, \frac{5}{2}, 3$.

If we take $\delta = -\frac{1}{2}$ we get the same roots.

Ex-(2), Solve, the equation $6x^3 - 11x^2 + 6x - 1 = 0$ whose roots are in H.P.

Soln ⇒ Let α, β, γ be the roots of the equation $6x^3 - 11x^2 + 6x - 1 = 0$. then

$\alpha + \beta + \gamma = \frac{11}{6}$ —— (I)

$\alpha\beta + \beta\gamma + \gamma\alpha = 1$ —— (II)

$\alpha\beta\gamma = \frac{1}{6}$ —— (III)

$\because \alpha, \beta, \gamma$ are in H.P. (Given)

Hence β is the H.M. between α & γ and

$\therefore \beta = \frac{2\alpha\gamma}{\alpha+\gamma} \Rightarrow \beta(\alpha+\gamma) = 2\alpha\gamma$

$\Rightarrow \alpha\beta + \beta\gamma = 2\alpha\gamma$

$\Rightarrow \alpha\beta + \beta\gamma + \alpha\gamma = 3\alpha\gamma$ —— (IV)

from (II) & (IV) $3\alpha\gamma = 1$ or $\gamma\alpha = \frac{1}{3}$ —— (V)

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Ex-(1) Solve the equation $x^3-9x^2+14x+24=0$, two of whose roots are in the ratio of $2:3$.

Soln $\rightarrow$

Let the roots of the equation be $2\alpha, 3\alpha, \beta$.

Then we have

$$2\alpha+3\alpha+\beta=9 \Rightarrow 5\alpha+\beta=9 \quad \text{--- (1)}$$

$$2\alpha\cdot 3\alpha+2\alpha\cdot\beta+3\alpha\cdot\beta=14 \Rightarrow 6\alpha^2+5\alpha\beta=14 \quad \text{--- (11)}$$

$$2\alpha\cdot 3\alpha\cdot\beta=-24 \Rightarrow \alpha^2\beta=-4 \quad \text{--- (111)}$$

Now, (11) $-\,5\alpha\times$(1) we have

$$19\alpha^2-45\alpha+14=0$$

We have $\alpha = \dfrac{45 \pm \sqrt{(45)^2-4\cdot 19\cdot 14}}{38}$

$$= \frac{45\pm 31}{38} = \frac{76}{38} \text{ or } \frac{14}{38} = 2 \text{ or } \frac{7}{19}$$

If $\alpha=2$ then from (1) $\beta=9-5\alpha=9-10=-1$

If $\alpha=\frac{7}{19}$ then from (1) $\beta=9-5\alpha=9-\frac{35}{19}=\frac{136}{19}$

But $\alpha=\frac{7}{19}$ and $\beta=\frac{136}{19}$ do not satisfy (111)

Hence we take $\alpha=2$ and hence the roots are 4, 6, and -1.

Ex-(2) Solve the equation $2x^3-15x^2+37x-30=0$ whose roots are in AP.

Soln $\rightarrow$ Let $\alpha-\delta, \alpha, \alpha+\delta$ be the roots of the equation

then, $(\alpha-\delta)+\alpha+(\alpha+\delta)=\frac{15}{2} \Rightarrow 3\alpha=\frac{15}{2} \therefore \alpha=\frac{5}{2}$ --- (1)

(2) Its observe that the number of terms in $\sum \alpha_1\alpha_2$ is $^nC_2$, for this is equal to the numbers of combinations of n roots taken two at a time. Similarly the number of terms in $\sum \alpha_1\alpha_2\alpha_3$ is $^nC_3$ and so on.

Working Rule →

Let $f(x) = 0$ be a (complex) equation of the nth degree and $\alpha_1, \alpha_2, \alpha_3 \ldots \alpha_n$ be its n roots. then

Sum of the roots $= -\dfrac{\text{Coefficient of } x^{n-1}}{\text{Coefficient of } x^n}$

Sum of the product of the roots taken two by two

$$= \frac{\text{Coefficient of } x^{n-2}}{\text{Coefficient of } x^n}$$

Sum of the product of the roots taken three by three $= -\dfrac{\text{Coefficient of } x^{n-3}}{\text{Coefficient of } x^n}$

Product of all the roots $= (-1)^n \cdot \dfrac{\text{Constant term}}{\text{Coefficient of } x^n}$

for Ex (1) let $ax^3+3bx^2+3cx+d=0$ be cubic equation and α, β, γ be its roots.

There we have, $\alpha+\beta+\gamma = \dfrac{-3b}{a}$

$$\alpha\beta+\alpha\gamma+\beta\gamma = \frac{3c}{a}$$

$$\alpha\beta\gamma = \frac{-d}{a}$$

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from (III), $\beta \cdot \frac{1}{3} = \frac{1}{6} \Rightarrow \beta = \frac{1}{2}$

Now, from (I), $\alpha + \frac{1}{2} + \gamma = \frac{11}{6} \Rightarrow \alpha + \gamma = \frac{11}{6} - \frac{1}{2} = \frac{4}{3}$ —— (VI)

Now solve (5) & (6) for α and γ.

we have $(\alpha - \gamma)^2 = (\alpha + \gamma)^2 - 4\alpha\gamma = \frac{16}{9} - \frac{4}{3} = \frac{4}{9}$

$\Rightarrow \alpha - \gamma = \pm \frac{2}{3}$ —— (VII)

Solving (VI) & (VII) we get

$\alpha = 1, \gamma = \frac{1}{3}$ or $\alpha = \frac{1}{3}, \gamma = 1$

Hence in either case, the roots are, $1, \frac{1}{2}$ & $\frac{1}{3}$

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(1)

Relation between the roots and coefficient

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Let $\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_n$ be the roots of the polynomial equation

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_{n-1}x + a_n = 0 \quad \text{---} (1)$$

Then we have the identity

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_{n-1}x + a_n$$

$$\equiv a_0(x-\alpha_1)(x-\alpha_2)(x-\alpha_3)\cdots(x-\alpha_{n-1})(x-\alpha_n)$$

$$= a_0\big[x^n - (\alpha_1+\alpha_2+\cdots+\alpha_n)x^{n-1} + (\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3+\cdots+\alpha_{n-1}\alpha_n)x^{n-2}$$

$$-(\alpha_1\alpha_2\alpha_3+\alpha_1\alpha_3\alpha_4+\cdots+\alpha_{n-2}\alpha_{n-1}\alpha_n)x^{n-3}$$

$$+ \cdots + (-1)^n\alpha_1\alpha_2\alpha_3\cdots\alpha_n\big]$$

Hence equating the coefficients of the like power of x from both sides we have

$$\Sigma\alpha_1 \equiv \alpha_1+\alpha_2+\alpha_3+\cdots+\alpha_n = \frac{a_1}{a_0}$$

$$\Sigma\alpha_1\alpha_2 \equiv \alpha_1\alpha_2+\alpha_1\alpha_3+\cdots+\alpha_{n-1}\alpha_n = \frac{a_2}{a_0}$$

$$\Sigma\alpha_1\alpha_2\alpha_3 \equiv \alpha_1\alpha_2\alpha_3+\alpha_1\alpha_2\alpha_4+\cdots+\alpha_{n-2}\alpha_{n-1}\alpha_n = \frac{-a_3}{a_0}$$

$$\cdots\cdots\cdots$$

$$\cdots\cdots\cdots$$

$$\alpha_1\alpha_2\alpha_3\cdots\alpha_n = (-1)^n\frac{a_n}{a_0}$$