MEASUREMENT OF TREND BY LEAST SQUARES METHOD

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“A set of data depending on the time is time series.”

-Kenny and keeping

Arrangement of statistical data in chronological order i.e., in accordance with occurrence of time is known as ‘Time Series'. Such series have a unique important place in the field of Economic, and Business Statistics since the series relating to price , consumption , and production of various commodities ; money in circulation , Bank Deposit and Bank clearing, sales and profits in a departmental store, agriculture and industrial production , national income and foreign exchange reserve , etc., are all time series spread over a long period of time. A time depicts the relationship between two variables, one of them being time. E.g., the population (Y=variable under consideration at time) of a country in different years (t=time).

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MEASUREMENT OF TREND

The various methods by which trend values can be determined are:

1. Free hand or Graphic Method
2. Semi-Average Method
3. Moving –Average Method
4. Least Square Method

Method of Least Squares Least Square is the method for finding the best fit of a set of data points. It minimizes the sum of the residuals of points from the plotted curve. It gives the trend line of best fit to a time series data. This method is most widely used in time series analysis.
Each point on the fitted curve represents the relationship between a known independent variable and an unknown dependent variable. This line is termed as the line of best fit from which the sum of squares of the distances from the points is minimized.

In this method the trend value Y' of the variable Y are computed so as to satisfy the conditions:

1. The sum of the deviations of Y from their corresponding trend values is zero.

i.e.,$\sum(Y-Y')=0$

2. The sum of the square of the deviations of the values of y from their corresponding trend values is the least.

i.e., $\sum(Y-Y')^2 =0$
is least.

Trend Line

Trend line (Y') is defined by the following equation:

$Y' = a + b X$

Where, Y = predicted value of the dependent variable

a = Y-axis intercept i.e. the height of the line above origin (when $X = 0$, $Y = a$)

b = slope of the line (the rate of change in Y for a given change in X)

When b is positive the slope is upwards, when b is negative, the slope is downwards

X = independent variable (in this case it is time)

To estimate the constants a and b, the following two Normal equations have to be solved simultaneously:

$\Sigma Y = n\,a + b\,\Sigma X$……………………(i)

$\Sigma XY = a\,\Sigma X + b\,\Sigma X2$………………..(ii)

To simplify the calculations, if the midpoint of the time series is taken as origin, then the negative values in the first half of the series balance out the positive values in the second half so that $\Sigma X = 0$. In this case, the above two normal equations will be as follows:

$\Sigma Y = n\,a$

$\Sigma XY = b\,\Sigma X2$

In such a case the values of a and b can be calculated as under:

Since, $\Sigma Y = n\ a$

$a = \Sigma Y/n$ ………………..(i)

Since, $\Sigma XY = b\backslash\Sigma X2$

$b = \Sigma XY/\Sigma X2$…………….(ii)

Example on Method of Least Squares

Calculate the trend value by the least squares from the data given below and estimate the sales for the year 2012.

year	2006	2007	2008	2009	2010
Sales of T.V(in 000)	12	18	20	23	27

Solution:

Total of 5 observations are there. So, the origin is taken at the Year 2008 for which X is assumed to be 0.

Year (t)	**Sale Y**	**X=t-2008**	**X^2**	**XY**	**Trend value Y'**
2006	12	-2	4	-24	13.0
2007	18	-1	1	-18	16.5
2008	20	0	0	0	20.0
2009	23	1	1	+23	23.5
2010	27	2	4	+54	27.0
N=5	$\sum Y=100$	$\sum X=0$	$\sum X^2=10$	$\sum XY=35$	$\sum Y'=100$

From the table we find that value of n is 5, value of ΣY is 100, value of ΣX is 0, value of ΣXY is 35, and value of ΣX^2 is 10 .

Substituting these values in the two given equations,

$a = \Sigma Y/n$ ………………(i)

$=100/5 = 20$

$a = 20$

$b = \Sigma XY/\Sigma X2$…………….(ii)

$=35/10$

$b = 3.5$

Trend equation is : $Y' = a + b X$

$Y' = 20 + 3.5 X$

Now, for 2012 the value of X would be 4, when X=+4, then trend value of Y or computed value of Y i.e. $Y'=20+3.5 \times 4 = 34$, thus the likely sale of T.V in 2012 will be 34 thousand.

Solution:

Total of 5 observations are there. So, the origin is taken at the Year 2005 for which X is assumed to be t-2005.

Year (t)	**Sales Y**	**X= t-2005**	**X^2**	**XY**	**Trend Value/Y '**
2006	12	1	1	12	13.0
2007	18	2	4	36	16.5
2008	20	3	9	60	20.0
2009	23	4	16	92	23.5
2010	27	5	25	135	27.0
N=5	$\sum Y=100$	$\sum X=15$	$\sum X^2=55$	$\sum XY=335$	$\sum Y'=100$

Trend line (Y') is defined by the following equation:

$Y' = a + b X$

To estimate the constants a and b, the following two Normal equations have to be solved simultaneously:

$\Sigma Y = n\,a + b\,\Sigma X$……………………(i)

$\Sigma XY = a\,\Sigma X + b\,\Sigma X2$………………..(ii)

From the table we find that value of n is 5, value of ΣY is 100, value of ΣX is 15 , value of ΣXY is 335, and value of ΣX^2 is 55.

Substituting these values in the two given equations,

$100 = 5a + 15b$……………………………..(i)
$335 = 15a + 55b$……………………………..(ii)

Multiplying eq. (i) by 3, we get:
$300 = 15a + 45b$ ……………………………(iii)

Subtract eq.(iii) from (ii), we get:
$35 = 10b$
Or $b=35/10=3.5$

Substitute the value of 'b' in equation (i), we get
$100 = 5a + 15 \times 3.5$
$100 = 5a + 52.5$
$5a = 100 - 52.5 = 47.5$
Therefore, $a = 47.5/5 = 9.5$ the equation of straight line now would be:
$Y' = 9.5 + 3.5X$

the equation of straight line now would be:

$Y' = 9.5 + 3.5\ X$

Now, for 2012 the value of X would be 7, when X= 7, then trend value of Y or computed value of Y i.e. $Y' = 9.5 + 3.5 \times 7 = 34$, thus the likely sale of T.V in 2012 will be 34 thousand. And Deviation between original value of Y and computed value of Y i.e. Y' is zero.

Years	Sale Y (Original value)	Trend value Y'	Deviation = Y-Y'
2006	12	13	-1
2007	18	16.5	1.5
2008	20	20	0
2009	23	23.5	-0.5
2010	27	27	0
N=5	$\sum Y=100$	$\sum Y=100$	$\sum(Y-Y')=0$